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Brief Paper

An Output Error Recursive Algorithm for Unbiased Identification in Closed Loop*

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Key Words—Identification algorithms; output-error identification; stability analysis; convergence analysis; passivity

Abstract—The problem of unbiased recursive identification of a plant model in closed-loop operation is considered. A particular form of an output error predictor for the closed loop is introduced. This allows one to derive a parameter estimation algorithm for the plant model that is globally asymptotically stable and asymptotically unbiased in the presence of noise. The paper presents a stability analysis in a deterministic environment and a convergence analysis in the stochastic environment. Both require a mild sufficient passivity condition to be satisfied. Simulations and real-time experiments on a flexible transmission illustrate the performances of the proposed algorithm. © 1997 Elsevier Science Ltd.

1. *Introduction*

The practical importance of plant model identification in closed loop has been recognized for many years, and a number of procedures that have been analyzed in detail are available (Gustavsson *et al.*, 1977; Ljung, 1987; Söderstrom and Stoica, 1989). However, in recent years a revival of interest for the methodology of plant model identification in closed-loop operation has occurred in the context of the iterative combination of identification in closed-loop and robust control re-design (Gevers, 1993; Van den Hof and Schrama, 1995). In this context, a new point of view has emerged, namely that the objective of plant model identification in closed loop is to get a better prediction for the closed loop via a better estimation of the plant model. While this idea was presented in an embryonic form in Ljung (1987) and theoretical tools for its development were available, it has not been explored in detail until recently, particularly in the area of recursive identification algorithms.

The above idea leads to the scheme of Fig. 1, where a predictor for the closed loop is built up as indicated, and the closed-loop prediction error is used to update the plant parameter estimates.

In this context, the problem of identification of a plant model in closed loop can be viewed in two different ways, which lead, however, to similar types of algorithms.

MRAS point of view. The true closed-loop system corresponds to a reference model, and a parallel adjustable system having a feedback configuration is built up. This adjustable feedback system contains a fixed controller and an adjustable model of the plant. The problem is to design a parameter adaptation algorithm assuring the global asymptotic stability of the closed-loop prediction error (this is a dual problem with respect to the classical model reference adaptive control problem).

Identification point of view (*Ljung*, 1987, *p.* 393).† Construct a re-parameterized adjustable predictor for the closed loop system in terms of a known fixed controller and of an adjustable plant model.

More specifically, in the case of recursive algorithms, the problem can be formulated as follows. Under the assumption that the controller is constant and known, identify a plant model such that

(i) global asymptotic stability is assured for any initial parameter estimates and initial error between the output of the true system and that of the closed loop predictor (in the absence of noise);

(ii) for an output disturbance independent of the external signal, an asymptotically optimal predictor for the closed-loop system is obtained;

(iii) under appropriate richness conditions, asymptotically unbiased estimation of the plant model parameters are obtained in the presence of noise.

In this paper, it is assumed that the input–output part of the plant to be identified belongs to the model set.

With respect to the various approaches for identification in closed loop (Söderstrom and Stoica, 1989), the technique presented can be viewed as a variation of the *direct approach* using a special kind of instrumental variable generated on line within the predictor itself (which requires knowledge of the controller and the access to the external signal).

The paper is organized as follows. In Section 2 the closed-loop recursive parameter estimation algorithms are presented. The stability analysis is presented in Section 3. The convergence properties in a stochastic environment are examined in Section 4. Simulation and real-time experiments are presented in Section 5. Conclusions are given in Section 6.

2. *The algorithms*

2.1. *The basic equations.* The objective is to estimate the parameters of the plant model described by the transfer operator:

$$H_p(q^{-1}) = \frac{q^{-d}B(q^{-1})}{A(q^{-1})}, \tag{1}$$

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† This specific reference was pointed out by one of the referees.

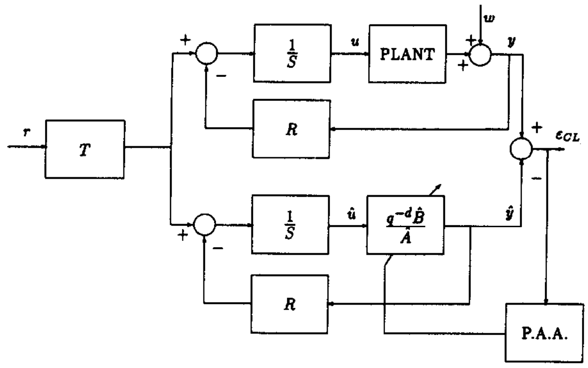

Fig. 1. Closed-loop output-error identification scheme.

where

$$B(q^{-1}) = b_1 q^{-1} + \ldots + b_{n_B} q^{-n_B} = q^{-1} B^*(q^{-1}), \quad (2)$$

$$A(q^{-1}) = 1 + a_1 q^{-1} + \ldots + a_{n_A} q^{-n_A} = 1 + q^{-1} A^*(q^{-1}). \quad (3)$$

The plant is operated in closed loop with an RST digital controller (without loss of generality) (Landau, 1990). The output of the plant operating in closed loop is given by (see Fig. 1)

$$\begin{aligned} y(t+1) &= -A^* y(t) + B^* u(t-d) + Aw(t+1) \\ &= \theta^{\mathrm{T}} \varphi(t) + Aw(t+1), \end{aligned} \quad (4)$$

where $u(t)$ is the plant input, $y(t)$ is the plant output, $w(t)$ is the output disturbance, a zero-mean stochastic process with finite moments independent of the reference signal, θ is the vector of coefficients of the transfer operator (1) and $\varphi(t)$ is the vector of measurements:

$$\theta^{\mathrm{T}} = [a_1 \ldots a_{n_A}, b_1 \ldots b_{n_B}], \quad (5)$$

$$\varphi^{\mathrm{T}}(t) \doteq [-y(t), \ldots, -y(t-n_A+1), u(t-d), \ldots, u(t-n_B+1-d)], \quad (6)$$

$$u(t) = -\frac{R}{S} y(t) + \frac{T}{S} r, \quad (7)$$

where r is the external excitation applied to the reference.

For a fixed value of the estimated parameters, the predictor of the closed loop (the design system) is described by

$$\hat{y}(t+1) = -\hat{A}^* \hat{y}(t) + \hat{B}^* \hat{u}(t-d) = \hat{\theta}^{\mathrm{T}} \phi(t), \quad (8)$$

where

$$\hat{\theta}^{\mathrm{T}} = [\hat{a}_1 \ldots \hat{a}_{n_A}, \hat{b}_1 \ldots \hat{b}_{n_B}], \quad (9)$$

$$\phi^{\mathrm{T}}(t) = [-\hat{y}(t) \ldots -\hat{y}(t-n_A+1), \hat{u}(t-d) \ldots \hat{u}(t-n_B+1-d)], \quad (10)$$

$$\hat{u}(t) = -\frac{R}{S} \hat{y}(t) + \frac{T}{S} r. \quad (11)$$

When the external excitation r_u is added to the output of the controller, $(T/S)r$ in (7) and (11) is replaced by r_u.

The closed-loop prediction (output) error is defined as

$$\varepsilon_{\mathrm{CL}}(t+1) = y(t+1) - \hat{y}(t+1). \quad (12)$$

The key observation is that the output of the closed-loop system given by (4) can be expressed as

$$\begin{aligned} y(t+1) &= \theta^{\mathrm{T}} \varphi(t) + Aw(t+1) \\ &= \theta^{\mathrm{T}} \phi(t) - \left(A^* + \frac{q^{-d} B^* R}{S}\right) \varepsilon_{\mathrm{CL}}(t) + Aw(t+1), \end{aligned} \quad (13)$$

and that

$$1 + q^{-1}\left(A^* + \frac{q^{-d} B^* R}{S}\right) = \frac{AS + q^{-d} BR}{S} = \frac{P}{S}, \quad (14)$$

where $P = AS + q^{-d} BR$ defines the poles of the true closed-loop system.

Subtracting (8) from (13) one obtains, after grouping the terms in $\varepsilon_{\mathrm{CL}}$ on the left hand side and passing both sides through S/P,

$$\varepsilon_{\mathrm{CL}}(t+1) = \frac{S}{P}(\theta - \hat{\theta})^{\mathrm{T}} \phi(t) + \frac{AS}{P} w(t+1). \quad (15)$$

Note that in the case of constant predictor parameters, since $\phi(t)$ and $w(t+1)$ are uncorrelated, an optimal predictor minimizing $E\{\varepsilon_{\mathrm{CL}}^2(t+1)\}$ is obtained for $\hat{\theta} = \theta$.

2.2. *The CLOE (closed-loop output-error) algorithm.* Now replacing the fixed predictor of the closed loop (8) by an adjustable predictor defined as

$$\text{a priori:} \quad \hat{y}^\circ(t+1) = \hat{\theta}^{\mathrm{T}}(t) \phi(t), \quad (16)$$

$$\text{a posteriori:} \quad \hat{y}(t+1) = \hat{\theta}^{\mathrm{T}}(t+1) \phi(t), \quad (17)$$

where $\hat{y}^\circ(t+1)$ and $\hat{y}(t+1)$ are the a priori and a posteriori outputs of the predictor, $\phi(t)$ contains the a posteriori predicted outputs and resulting inputs, and $\hat{\theta}(t)$ is the adjustable parameter vector, one gets the a priori and a posteriori prediction errors $\varepsilon_{\mathrm{CL}}^\circ(t+1)$ and $\varepsilon_{\mathrm{CL}}$ respectively as

$$\varepsilon_{\mathrm{CL}}^\circ(t+1) = y(t+1) - \hat{y}^\circ(t+1), \quad (18)$$

$$\varepsilon_{\mathrm{CL}}(t+1) = y(t+1) - \hat{y}(t+1). \quad (19)$$

The equation for the a posteriori prediction error becomes, in the deterministic case ($w(t) \equiv 0$),

$$\varepsilon_{\mathrm{CL}}(t+1) = \frac{S}{P}[\theta - \hat{\theta}(t+1)]^{\mathrm{T}} \phi(t). \quad (20)$$

This equation has the typical form encountered in discrete-time MRAS and PLR (pseudo-linear regression) recursive identification methods including the output error for open-loop identification. Therefore it is reasonable to consider a PAA of the form used in this case. A general form of such algorithm and the related stability analysis in connection with an equation of the form (20) can be found in Landau (1980, Theorem 2.1). Such a PAA has the form

$$\hat{\theta}(t+1) = \hat{\theta}(t) + F(t) \phi(t) \varepsilon_{\mathrm{CL}}(t+1), \quad (21)$$

$$F^{-1}(t+1) = \lambda_1(t) F^{-1}(t) + \lambda_2(t) \phi(t) \phi^{\mathrm{T}}(t), \quad (22)$$

$$0 < \lambda_1(t) \leq 1, \quad 0 \leq \lambda_2(t) < 2,$$

$$F(0) > 0, \quad F^{-1}(t) > \alpha F^{-1}(0), \quad 0 < \alpha < \infty,$$

$$F(t+1) = \frac{1}{\lambda_1(t)}\left[F(t) - \frac{F(t)\phi(t)\phi^{\mathrm{T}}(t)F(t)}{\lambda_1(t)/\lambda_2(t) + \phi^{\mathrm{T}}(t)F(t)\phi(t)}\right], \quad (23)$$

$$\varepsilon_{\mathrm{CL}}(t+1) = \frac{\varepsilon_{\mathrm{CL}}^\circ(t+1)}{1 + \phi^{\mathrm{T}}(t)F(t)\phi(t)} = \frac{y(t+1) - \hat{y}^\circ(t+1)}{1 + \phi^{\mathrm{T}}(t)F(t)\phi(t)}. \quad (24)$$

$\lambda_1(t)$ and $\lambda_2(t)$ allow one to obtain various profiles for the adaptation gain (for details, see Landau, 1980). Equation (24) is obtained from (18), (19) and (21), observing that

$$\begin{aligned} \varepsilon_{\mathrm{CL}}^\circ(t+1) - \varepsilon_{\mathrm{CL}}(t+1) &= [\hat{\theta}(t+1) - \hat{\theta}(t)]^{\mathrm{T}} \phi(t) \\ &= \phi^{\mathrm{T}}(t) F(t) \phi(t) \varepsilon_{\mathrm{CL}}(t+1). \end{aligned} \quad (25)$$

2.3. *The F-CLOE (filtered closed-loop output-error) algorithm.* One can also derive a 'filtered' algorithm. Define

$$\phi_f(t) = \frac{S}{\hat{P}} \phi(t), \quad (26)$$

where

$$\hat{P} = \hat{A}S + q^{-d}\hat{B}R \quad (27)$$

is an estimation of the closed-loop poles based on an estimation of A and B (obtained, for example, from an open-loop identification). Neglecting the non-commutativity of the time-varying operators, (20) can be rewritten as

$$\begin{aligned}\varepsilon_{CL}(t+1) &= \frac{S}{P}[\theta - \hat{\theta}(t+1)]^T \frac{\hat{P}}{S}\frac{S}{\hat{P}}\phi(t) \\ &= \frac{S}{P}\frac{\hat{P}}{S}[\theta - \hat{\theta}(t+1)]^T \phi_f(t) \\ &= \frac{\hat{P}}{P}[\theta - \hat{\theta}(t+1)]^T \phi_f(t), \end{aligned} \quad (28)$$

which allows one to derive a recursive parameter estimation with a filtered observation vector. One uses the parameter adaptation algorithm of (21)–(24) in which $\phi(t)$ is replaced by $\phi_f(t)$.

Note that an exact algorithm can be derived, but the formula (24) becomes more complicated.

3. *Stability analysis*

The results of the stability analysis are presented in the following theorem.

Theorem 3.1. Assuming that the closed-loop system is stable, the recursive parameter estimation algorithm given by (21)–(24) assures

$$\lim_{t\to\infty} \varepsilon_{CL}(t+1) = 0, \quad (29)$$

$$\lim_{t\to\infty} \varepsilon^{\circ}_{CL}(t+1) = 0, \quad (30)$$

$$\|\phi(t)\| < C, \quad 0 < C < \infty, \quad \forall t \quad (31)$$

for all initial conditions $\hat{\theta}(0)$, $\varepsilon^{\circ}_{CL}(0)$ and $\phi(0)$ if

$$H'(z^{-1}) = \frac{S(z^{-1})}{P(z^{-1})} - \frac{\lambda}{2}, \quad \sup_t \lambda_2(t) \le \lambda < 2, \quad (32)$$

is a strictly positive real transfer function.

Proof. The form of the equation (20) for the a posteriori error, and the equations (21) and (22) of the parameter adaptation algorithm, allows one to use the results of Landau (1980), and it follows immediately that the condition (32) implies (29).

It remains to show that (30) and (31) hold. To prove (30) one has to show that $\phi(t)$ is bounded. However, the components of $\phi(t)$ are $\hat{y}(t-i) = y(t-i) - \varepsilon_{CL}(t-i)$, $i = 0, \ldots, \max(n_{A-1}, n_{B-1+d})$, eventually filtered by R/S. Assuming the asymptotic stability of S and the boundness of the system output $y(t)$ and of the external excitation $r(t)$, it results that $\phi(t)$ will be bounded if $\varepsilon_{CL}(t)$ is bounded (since $\hat{y}(t) = y(t) - \varepsilon_{CL}(t)$). But this is indeed true because of the result of Theorem 2.1 in Landau (1980). □

Remarks.

(i) A filter D on the error can be also introduced. In this case the condition (32) will be of the form $DS/P - \frac{1}{2}\lambda$ is strictly positive real.

(ii) For the case of the 'filtered' algorithm, the positive real condition (32) is replaced by

$$H'(z^{-1}) = \frac{\hat{P}(z^{-1})}{P(z^{-1})} - \frac{\lambda}{2} \quad \text{is strictly positive real,} \quad (33)$$

which is much milder if a reasonable estimated model is available (obtained, for example, from an open-loop identification).

(iii) The design of the controller influences the stability condition.

(iv) Unstable plants can be identified if a stabilizing controller with stable S is used.

Relaxation of the positive real condition. The positive real condition (32) or (33) can be relaxed using the method proposed in Tomizuka (1982). This consists in replacing the integral-type PAA of (21) by an 'integral + proportional' type PAA (for details see Landau, 1979)

$$\hat{\theta}(t+1) = \hat{\theta}_I(t+1) + \hat{\theta}_P(t+1), \quad (34)$$

$$\hat{\theta}_I(t+1) = \hat{\theta}_I(t) + F(t)\phi(t)\varepsilon_{CL}(t+1), \quad (35)$$

$$\hat{\theta}_P(t+1) = F_P(t)\phi(t)\varepsilon_{CL}(t+1), \quad F_P(t) > 0, \quad (36)$$

such that

$$\phi^T(t)[\tfrac{1}{2}F(t) + F_P(t)]\phi(t) \ge K_{\min}, \quad (37)$$

where $K_{\min}$ is the minimum value of feedback gain that makes

$$H_K(z^{-1}) = \frac{S/P - \frac{1}{2}\lambda}{1 + K(S/P - \frac{1}{2}\lambda)} \quad (38)$$

strictly positive real (this problem always has a solution for discrete-time transfer functions).

4. *Convergence analysis in a stochastic environment*

One of the objectives of closed-loop identification is to obtain asymptotic unbiased estimates in the presence of noise on the plant output. We shall use for this analysis the ODE approach (Ljung, 1977) and a specific result for a class of parameter-estimation algorithms (Dugard and Landau, 1980).

The equation of the a posteriori prediction error in the presence of noise is

$$\varepsilon_{CL}(t+1) = \frac{S}{P}[\theta - \hat{\theta}(t+1)]^T\phi(t) + \frac{AS}{P}w(t+1). \quad (39)$$

One has the following result.

Theorem 4.1. Consider the parameter-estimation algorithm given by (21)–(24) with $\lambda_1(t) = 1$. Define

$$\phi(t, \hat{\theta}) \triangleq \phi(t)|_{\hat{\theta}(t) = \hat{\theta} = \text{const}},$$

$$\varepsilon_{CL}(t+1, \hat{\theta}) \triangleq \varepsilon_{CL}(t+1)|_{\hat{\theta}(t) = \hat{\theta} = \text{const}},$$

$$D_s \triangleq \{\hat{\theta} : \hat{A}(z^{-1})S(z^{-1}) + z^{-d}\hat{B}(z^{-1})R(z^{-1}) = 0 \Rightarrow |z| < 1\}.$$

- Assume that $\hat{\theta}(t)$ generated by the algorithm belongs infinitely often to the domain D_s for which the stationary processes $\phi(t, \hat{\theta})$ and $\varepsilon_{CL}(t+1, \hat{\theta})$ can be defined.
- Assume that $w(t)$ is a zero-mean stochastic process with finite moments independent of the reference sequence $r(t)$.

If

$$H'(z^{-1}) = \frac{S(z^{-1})}{P(z^{-1})} - \frac{\lambda}{2}, \quad \sup_t \lambda_2(t) \le \lambda < 2, \quad (40)$$

is a strictly positive real discrete transfer function then

$$\text{Prob}\left\{\lim_{t\to\infty} \hat{\theta}(t) \in D_c\right\} = 1, \quad (41)$$

where $D_c = \{\hat{\theta} : \phi^T(t, \hat{\theta})(\theta - \hat{\theta}) = 0\}$. If, furthermore, $\phi^T(t, \hat{\theta})(\theta - \hat{\theta}) = 0$ has a unique solution (richness condition) then the condition that $H'(z^{-1})$ given by (40) be strictly positive real implies that

$$\text{Prob}\left\{\lim_{t\to\infty} \hat{\theta}(t) = \theta\right\} = 1. \quad (42)$$

Proof. From (39), it results that, for $\hat{\theta}(t) = \theta$,

$$\varepsilon_{CL}(t+1, \hat{\theta}) = \frac{S}{P}(\theta - \hat{\theta})^T\phi(t, \hat{\theta}) + \frac{AS}{P}w(t+1). \quad (43)$$

The form of (43) and the independence between $\phi(t, \hat{\theta})$ and $w(t+1)$ allows one to straightforwardly apply the results of Dugard and Landau (1980) or Ljung and Söderstrom (1983) for the case $\lambda_2 = \text{constant}$. However, these results are also applicable for the case $\lambda_2(t)$ by replacing in the corresponding proofs $\lambda_2(t)$ by $\lambda + [\lambda_2(t) - \lambda]$.

Remarks.

(i) The same analysis applied to the 'filtered' algorithm requires that

$$H'(z^{-1}) = \frac{\hat{P}(z^{-1})}{P(z^{-1})} - \frac{\lambda_2}{2} \tag{44}$$

be strictly positive real.

(ii) In the F-CLOE algorithm one can replace in the data filter $\hat{P}(q^{-1})$ by $\hat{P}(q^{-1}, t)$ (the current estimate of closed-loop poles), one gets an RPEM (recursive prediction error method)-type algorithm (Ljung, 1987), but for this method only local results can be derived. Therefore an initialization by another method and stability tests should be added.

5. *Simulation and experimental results*

5.1. *Simulation results.* The objective of the simulation is to show the behaviour of the proposed algorithms when the positive real condition for convergence is violated. In addition, an unstable plant is considered in order to show that the stability of the plant is not a necessary condition for the stability of the algorithms.

For this simulation, the parameters of the system are

$$A(q^{-1}) = 1 - 2.5q^{-1} + 0.2q^{-2},$$

$$B(q^{-1}) = q^{-1}(1 + 0.5q^{-1}).$$

The output disturbance $w(t)$ is chosen as

$$w(t) = \frac{C(q^{-1})}{A(q^{-1})} e(t),$$

where $e(t)$ is a zero-mean uniformly distributed white noise and

$$C(q^{-1}) = 1 + 0.5q^{-1} + 0.5q^{-2}.$$

The open-loop system, which is in fact unstable, can be stabilized using a unit feedback ($R = S = 1$). The characteristic polynomial of the closed-loop system is given by

$$P = 1 - 1.5q^{-1} + 0.7q^{-2}, \tag{45}$$

which leads to a non-positive-real discrete-time transfer function for S/P.

For identification of the plant model in closed loop, a pseudorandom binary sequence (PRBS) generated by a 7-bit shift register and a clock frequency of $\frac{1}{2}f_s$ (sampling frequency $f_s = 1$) is considered as reference signal. The noise signal ratio at the output of the closed-loop system is about 10% in terms of variance.

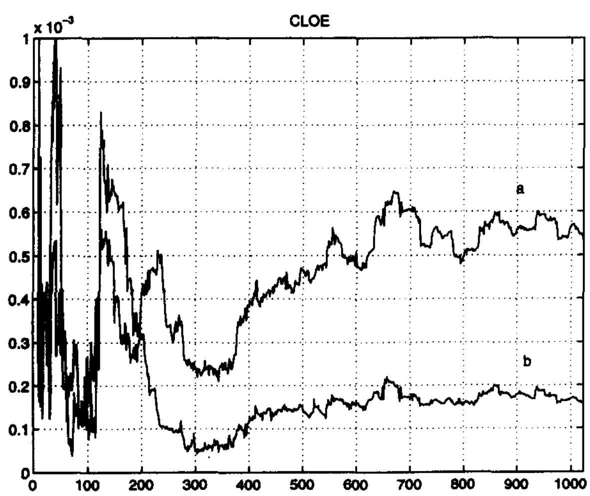

Fig. 2. Evolution of the parametric distance for the CLOE algorithm: (a) using integral adaptation; (b) using integral + proportional adaptation.

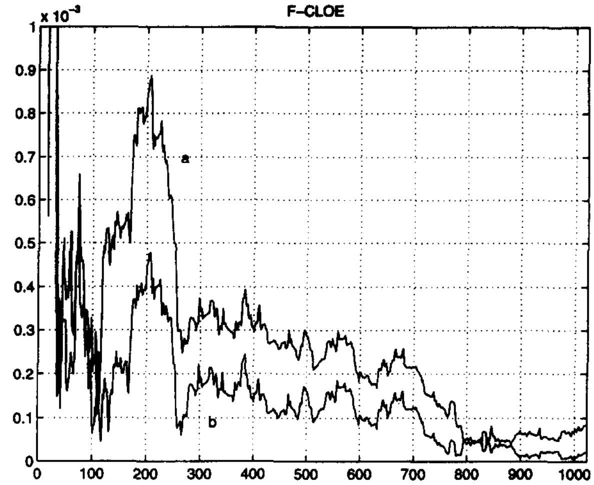

Fig. 3. Evolution of the parametric distance for the F-CLOE algorithm: (a) using an on-line estimation of $\hat{P}(t) = \hat{A}(t)S + q^{-d}\hat{B}(t)R$; (b) using the exact S/P.

In order to study the convergence of the algorithms, the parametric distance defined by

$$D(t) = \left\{ \sum_{i=1}^{n_A} [a_i - \hat{a}_i(t)]^2 + \sum_{i=0}^{n_B} [b_i - \hat{b}_i(t)]^2 \right\}^{1/2} \tag{46}$$

may be used as a criterion. Figures 2 and 3 show the evolutions of the parametric distance for CLOE and F-CLOE methods. One can observe that the CLOE method with integral parameter adaptation (curve (a)) does not converge (it does not diverge either) when S/P is not a positive real function. This algorithm will converge to the true values (curve(b)) using the modification explained in Section 4 ($F_P = 0.0015I$), which relaxes the positive real condition. One also sees that the F-CLOE method converges to the true values. It should be noticed that the true parameters of the plant are used to compute an estimation of the characteristic polynomial $\hat{P}$ in the F-CLOE algorithm for the curve (b) and an on-line estimation of the polynomial $\hat{P}(t) = \hat{A}(t)S + q^{-d}\hat{B}(t)R$ is used for the curve (a). However, this example clearly justifies the fact that the positive real condition on S/P can be relaxed using some a priori information about the plant.

5.2. *Experimental evaluation.* The experimental device is depicted in Fig. 4. It consists of a flexible transmission formed by a set of three pulleys coupled by two very elastic belts. The system is controlled by a PC via an I/O board. The sampling frequency is 20 Hz.

The system identification is carried out in open loop with a PC using PIM/TR identification software (Adaptech, 1995). The output error with extended prediction model

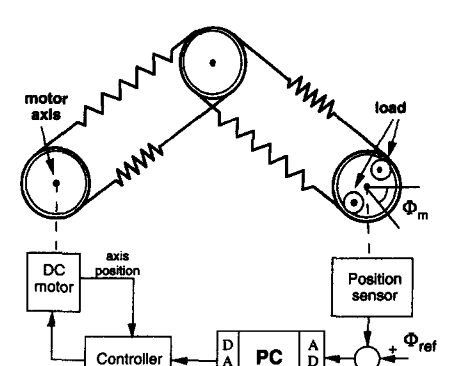

Fig. 4. Block diagram of the flexible transmission.

(Landau 1990) provided the best results in terms of statistical model validation. The model obtained is

$$A = 1 - 1.3528q^{-1} + 1.5502q^{-2} - 1.2798q^{-3} + 0.9115q^{-4},$$

$$B = 0.4116q^{-1} + 0.524q^{-2}, \quad d = 2.$$

The main characteristics of the system are two very oscillatory modes, an unstable zero and a time delay of two sampling periods. A controller for this system is computed by the pole-placement method with PC-REG software (Adaptech, 1995). The controller is designed in order to obtain two dominant poles with the same frequency as the first mode of the open-loop model but with a damping factor of 0.8. The precompensator $T(q^{-1})$ is chosen to obtain unit closed-loop gain. The parameters of the RST controller are as follows:

$$R(q^{-1}) = 0.4526 - 0.4564q^{-1} - 0.6857q^{-2} + 1.0955q^{-3} - 0.1449q^{-4},$$

$$S{-}q^{-1}) = 1 + 0.2345q^{-1} - 0.8704q^{-2} - 0.4474q^{-3} + 0.0833q^{-4},$$

$$T(q^{-1}) = 0.2612.$$

The above controller has been implemented on the real platform using PCREG-TR software (Adaptech, 1995). The identification of the plant in closed loop is carried out using the CLOE method. A PRBS generated by a 7-bit shift register and a clock frequency of $\frac{1}{2}f_s$ is considered as reference signal. The parameters of the plant model identified in closed loop are given by

$$A = 1 - 1.3155q^{-1} + 1.5382q^{-2} - 1.2852q^{-3} + 0.9406q^{-4},$$

$$B = 0.7024q^{-1} + 0.1820q^{-2}, \quad d = 2.$$

The frequency response of the model identified in closed loop is compared with that of the open-loop model in Fig. 5. The first oscillatory mode is almost identical in the two models, while the second identified mode is rather different.

In order to validate the identified model, the real achieved closed-loop poles (which can be obtained by identification of the whole closed-loop system with standard open-loop identification methods) and the computed ones (using the plant model identified by CLOE method and the RST controller) are given in Fig. 6. It is observed that the closed-loop poles computed using the proposed method and the real ones are almost superimposed, particularly at low frequencies, whereas the poles computed using the open-loop identified model are very far from the real closed-loop poles. It can be concluded that the plant model identified in closed loop (using the CLOE algorithm) gives a much better prediction for the behaviour of the closed-loop system than the model identified in open loop.

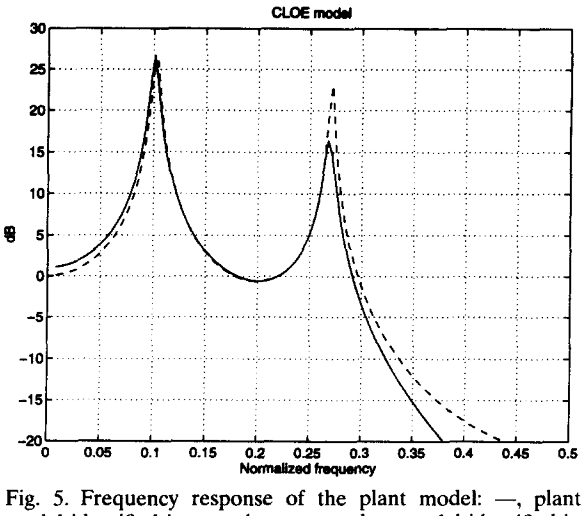

Fig. 5. Frequency response of the plant model: —, plant model identified in open loop; - - -, plant model identified in closed loop using the CLOE method.

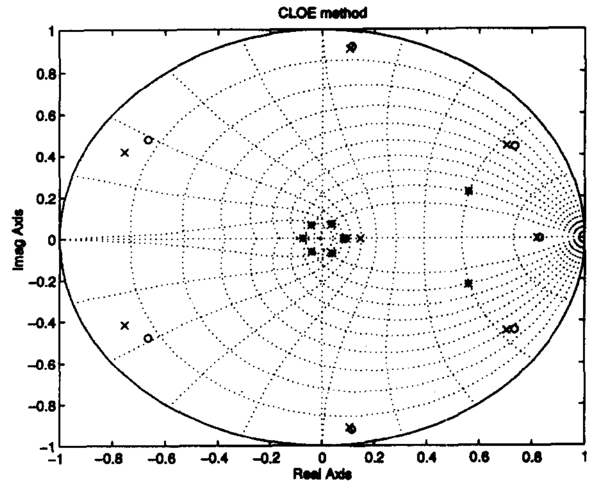

Fig. 6. Closed-loop poles chart: ○, real achieved closed-loop poles; ×, poles computed using the closed-loop identified model and the controller; *, poles computed using the open-loop identified model and the controller.

6. *Conclusions*

A new algorithm for unbiased estimation in closed loop has been presented. It belongs to the class of output error algorithms, and can be interpreted as a recursive pseudolinear regression.

Sufficient conditions for stability in a deterministic environment and convergence in a stochastic environment are related to a positive real condition on a sensitivity-type function. This condition can be relaxed by data filtering or adding a proportional adaptation.

Simulation and experimental results have confirmed the theoretical analysis.

The identified models can be validated using statistical tests (uncorrelation of ϕ and ε_{CL}), as well as by checking the closeness of the computed closed-loop poles with the true closed-loop poles, which can be obtained through identification of the closed loop.

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References

Adaptech (1995) *PIM+/TR (includes, PC-REG and PCREG/TR) System Identification and Control Software.* Adaptech, 4 Rue du Tour de l'eau, 38400 Saint Martin d'Heres, France.

Dugard, L. and Landau, I. D. (1980) Recursive output error identification algorithms. *Automatica* **16**, 443–462.

Gevers, M. (1993) Towards a joint design of identification and control? In *Essays on Control: Perspectives in the Theory and its Applications*, ed. H. L. Trentelman and J. Willems, pp. 111–152. Birkhäuser, Boston.

Gustavsson, I., Ljung, L. and Söderstrom, T. (1977) Identification of processes in closed loop—identifiability and accuracy aspects. *Automatica* **13**, 59–75.

Landau, I. D. (1979) *Adaptive Control—The Model Reference Approach.* Marcel Dekker, New York.

Landau, I. D. (1980) An extension of a stability theorem applicable to adaptive control. *IEEE Trans. Autom. Control* **AC-25**, 814–817.

Landau, I. D. (1990) *System Identification and Control Design.* Prentice-Hall, Englewood Cliffs, NJ.

Ljung, L. (1977) On positive real transfer function and the convergence of some recursive schemes. *IEEE Trans. Autom. Control* **AC-22**, 539–551.

Ljung, L. (1987). *System Identification—Theory for the User.* Prentice-Hall, Englewood Cliffs, NJ.
Ljung, L. and Söderstrom, T. (1983) *Theory and Practice of Recursive Identification.* MIT Press, Cambridge, MA.
Söderstrom, T. and Stoica, P. (1989) *System Identification.* Prentice-Hall, Hemel Hempstead, U.K.
Tomizuka, M. (1982) Parallel MRAS without compensation block. *IEEE Trans. Autom. Control* **AC-27,** 505–506.
Van den Hof, P. and Schrama, R. (1995) Identification and control—closed-loop issues. *Automatica* **31,** 1751–1770.